

# Electromagnetic Dipole Response as a Test of the $^{11}\text{Li}$ g.s. Structure and the $n$ - $^9\text{Li}$ Interaction

Russian-Nordic-British Theory (RNBT) collaboration

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## Abstract

The electric dipole response of the halo nucleus  $^{11}\text{Li}$  is calculated in a hyperspherical three-body formulation, and is studied as a function of the interaction employed for  $n$ - $^9\text{Li}$  to reflect the Pauli principle. Strength concentrations at lower energies are found but no narrow resonances. Only one possible scenario of  $^{11}\text{Li}$  structure is in close correspondence with MSU and RIKEN experimental data.

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Very recently, the experimental data for the electric dipole strength distribution  $dB(\mathcal{E}1, E)/dE$  have become available from MSU [1] for the two-neutron halo nucleus  $^{11}\text{Li}$  at excitation energies  $E \leq 2$  MeV. The Michigan fragmentation experiment for  $^{11}\text{Li} + \text{Pb}$  was carried out at 28A MeV. While previous data on inclusive variables such as geometrical properties and momentum distributions for individual break-up fragments have proven unable to discriminate between quite different wave functions for the bound state of  $^{11}\text{Li}$  [2, 3, 4], we will show that the dipole response function sheds some light on this issue. We remind the reader about the peculiar 3-body Borromean structure of  $^{11}\text{Li}$ : since there is only one bound state (weakly bound at about 300 keV) and none of the binary subsystems are able to form bound states, the asymptotic conditions of the problem are pure 3-body asymptotics.

Fig. 1 shows the deconvoluted experimental Michigan data which peaks at around 0.6 MeV, with energy being measured from the break-up threshold. The same peak position was reported[5] for the RIKEN experiment with the same reaction at 42A MeV. Fig. 1 also shows previous theoretical results, that of the simple point-dineutron cluster model of Bertulani and Bauer (BB)[6, 7], and the more realistic model of Esbensen and Bertsch (ESB)[8]. We notice that the calculations, which nearly coincide, are shifted to low energies compared with the data. In the BB cluster model the energy dependence is simply  $E_s^{1/2}E^{3/2}/(E + E_s)^4$ , where  $E_s \approx 0.3$  MeV is the separation energy of the two halo neutrons. This distribution peaks at  $E = \frac{3}{5}E_s$ , giving a linear dependence on the separation energy. The Green's function method of ESB employs a  $\delta$ -type  $nn$  interaction.

This letter investigates three limiting cases of plausible dynamics for the halo neutrons of  $^{11}\text{Li}$ , corresponding to three tentative prescriptions for treating the Pauli Principle within a strict 3-body formulation. Expansion of hyperspherical harmonics (HH) is used [9], a natural procedure for this Borromean system.

We have recently [10, 11, 12] and with considerable success, carried out within the same HH framework bound state and continuum calculations for  $^6\text{He}$ , also a Borromean nucleus, which shares many properties with  $^{11}\text{Li}$ . Contrary to  $^{11}\text{Li}$ , however, sufficient information is available on the binary  $n$ -core interaction, and this makes physically reliable calculations possible. Both the  $0^+$  ground state and the known  $2^+$  resonance at an excitation energy of 1.8 MeV were reproduced by our calculations. However, although these calculations gave strength concentrations at low continuum energies, they showed no narrow resonances neither in the phase shifts nor in the calculated strength functions for electric dipole excitations of  $^6\text{He}$ . A parallel study of  $^{11}\text{Li}$  is hampered by lack of information on the  $n-^9\text{Li}$  channel, although calculations can technically be carried out[3, 2].

The dipole response has the form

$$B(\mathcal{E}1; 0_{\text{g.s.}}^+ \rightarrow 1^-(E)) = |\langle \mathcal{E}1^-(E) | T_1^\mathcal{E} | 0_{\text{g.s.}}^+ \rangle|^2,$$

where  $T_{1M}^\mathcal{E} = \sum_i e_i r_i Y_{1M}(\hat{r}_i)$  is the corresponding intrinsic 3-body cluster dipole operator with coordinates referred to the c.m. of  $^{11}\text{Li}$ , and hence free from spurious c.m. motion. Note that the operator explores both the  $1^-$  structure of the continuum as well as the structure of the  $0^+$  ground state. (We leave the assumed  $^9\text{Li}(3/2^-)$  core out of all our calculations and discussion.) With previous experience from a similar calculation for  $^6\text{He}$  [12], we expect that replacing six-dimensional plane waves in the  $n - n$  and  $(nn) - ^9\text{Li}$  coordinates by correlated states from strict 3-body continuum calculations will shift the continuum strength to lower energies. Fig. 2 shows that this expectation is borne out in all cases. Notice that leaving out continuum correlations for the ESB case moves the peak into the position of the data, but that it now falls significantly below the data in strength.

We follow [2, 13] and consider three choices, referred to as the H, WS-P and WS-F scenarios. For all cases considered, the  $nn$  interaction is that of Gogny-Pires-de Tournell (GPT)[14] and includes repulsion at small distances, as well as spin-orbit and tensor forces. The Pauli principle is included by introducing repulsive ‘Pauli cores’ from the required combination(s) of central and spin-orbit forces. As discussed in previous papers [3, 2], all scenarios give a binding energy for  $^{11}\text{Li}$  of  $\sim 0.3$  MeV, i.e. within experimental error, and give [4] very similar single-particle densities. As in the  $^6\text{He}$  case, investigations of the three-body scattering amplitudes [15] showed no narrow dipole resonances states for any of the scenarios. These three scenarios give however very different ground-state wave functions, as exhibited in table 1.

In the WS-F (Woods-Saxon-Forbidden) scenario, the spin-orbit force is assumed to play a leading role. We assume that there is a  $0p_{1/2}$  resonance in  $n + ^9\text{Li}$  scattering ([16, 17, 18]), with the  $0p_{3/2}$  existing as a bound state of  $^9\text{Li}$  (B.E. = 4.1 MeV). The  $^9\text{Li}$  core is then taken to have a full level of  $0p_{3/2}$  neutrons, and the Pauli principle was taken into account by pure repulsive potential in the occupied  $0s$  and  $0p_{3/2}$  states of neutron-core motion. The resulting  $^{11}\text{Li}$  wave function (WF) will be predominantly  $(0p_{1/2})^2$ , which corresponds (for harmonic oscillator orbitals) to a linear combination of 33%  $^1S_0$  and 67%  $^3P_1$  states of neutron-neutron motion. This implies that the  $^{11}\text{Li}$  ground state has a rather small probability of a di-neutron configuration, starting at 33% and increasing to 43% (see table 1) when correlations are included.

In the WS-P (with P for pairing) scenario, we follow [19] by taking the pairing rather

than the spin-orbit forces as dominant in  $^{11}\text{Li}$ . A Cohen-Kurath type of calculation for  $^9\text{Li}$  gives [19] a ground state that is 93% of the symmetry  $[f] = [432]$ , which can couple *only* with the spatially symmetric ( $[f] = [2]$ ) neutron pair to form  $^{11}\text{Li}$  with a closed shell structure. The valence neutrons are now almost entirely in  $(l_{nn} = 0, S = 0)$  relative motion, with the  $S = 1$  configuration blocked by the core neutrons. The spin-orbit force being damped in the interior so that it does not destroy the coherence of the state which is approximately  $\sqrt{2/3}(0p_{3/2})^2 + \sqrt{1/3}(0p_{1/2})^2$ . We include this effect in our model by having no neutron-core spin-orbit forces, taking into account only the  $S = 0$  configurations in the  $0p$  shell. As in WS-F case we use a Pauli repulsive  $s$ -wave core- $n$  interaction.

The third (H) scenario has no explicit Pauli terms, but simply employs the shallow Gaussian potential of [20, 3], which does not support any occupied orbits. Thus the mean fields for the halo and core neutrons differ substantially. The halo neutrons may now be largely in  $0s$  motion relative to the core, and only partially blocked by Pauli orthogonality with the core neutrons because the core and valence radii are different. This calculation also reproduces the binding energy and r.m.s. matter radius of the  $^{11}\text{Li}$  ground state and gives the largest value of hyper-radial moments  $\langle |\rho^4| \rangle$  compared to  $\langle |\rho^2| \rangle^2$  (see Table 1) as an expression of the softness of the  $^{11}\text{Li}$  halo system.

In all scenarios, the ground state of  $^{11}\text{Li}$  has a closed neutron shell:  $(0s)^2$  in the H case, and  $(0s)^2(0p)^6$  in the WS-P and WS-F cases. Thus, to form a  $1^-$  excitation, a neutron will have to be excited into the next shell of opposite parity, implying an energy which is  $\hbar\omega \approx 4$  MeV in a standard estimate for dipole excitations.

For halo nuclei, however, the neutrons are very near threshold, and simple estimates based on  $\hbar\omega \approx 4$  MeV may no longer be correct. Recently, for example, it was argued in a self-consistent density-functional method [21], and also in a two-body cluster calculation [7], that the centre of gravity of strength functions should be concentrated closer to zero energy as the valence level approaches threshold. To answer this question, we have calculated continuum distributions in a 3-body model, which should include the effect of such shifts as a 3-body threshold feature.

In all scenarios, we obtain a satisfactory convergence for the  $\mathcal{E}1$  strength in the energy region below 10 MeV. However, for energies greater than about 4 or 5 MeV (the threshold for  $^9\text{Li}$  excitation) more complicated mechanisms come into play, and our analysis assuming an inert core is no longer physically valid. We must also ensure convergence in the radial integrations. Since the effective range (a product of two wave functions and a radial operator) of the dipole operator is  $\sim 40$  fm, to obtain the response without erroneous contributions from artificial low lying additional structures, we have extended

our calculations out to 150–200 fm.

The results for continuum final states (using the same 3-body hamiltonian as for the g.s.) are shown in Fig. 1, compared with the MSU data. We show also the results (ESB) of the Green function method[8], and the distributions (BB) of the 2-body cluster model[6]. The WS-P and WS-F distributions, as expected from simple estimates, are very broad. The continuum RPA results[22] are similar to the WS-P curve. Only the H scenario resembles the MSU data.

The curves peak at different energies. The position of the BB peak has already been discussed. In the WS-P model, qualitatively speaking, we have a purely repulsive  $s$ -wave  $n$ -core interaction, lifting up the levels in the next shell, and in WS-F there is also a  $p_{3/2}$  repulsion, additionally pushing the levels to higher energies. The curves also approach  $E = 0$  in different ways. In the 2-body BB[7] point dineutron cluster model an  $E^{3/2}$  behaviour is obtained for the low-energy  $\mathcal{E}1$  response. Since the g.s. WF is concentrated in the asymptotic tail, it is possible to estimate this quantity in the three-body case, which gives  $\sim E^3$  for the threshold behaviour. Both models give  $\sim E^{-5/2}$  when  $E$  is much larger than the binding energy.

In the calculation of continuum WFs the ground state core- $n$  potential and realistic GPT  $nn$  interaction were used. If we replace this  $nn$  interaction by a simple central Gaussian (which however reproduces  $n - n$  low energy phase shift data), all curves shown in figs. 1 & 2 are changed in the H case by less than the line width, and by only a few percent in the WS cases. Only the core- $n$  and central part of the  $nn$  interaction are decisive for the  $^{11}\text{Li}$  dynamics.

In our calculations of the electric dipole response of  $^{11}\text{Li}$  with 3-body wave functions for both the continuum and g.s., we summarise our results as follows:

i) Although the strength is concentrated at lower energies, resembling resonant behaviour, no narrow resonances are found in the three-body scattering amplitudes [15], implying a wide spreading of dipole states into the continuum;

ii) Our calculations support qualitatively the conclusion that the enhancement of dipole strength at low energy is mainly due to the halo structure of the g.s. That is, the kinematical (threshold effect) enhancement is due to the large moments of the mass distribution, and to the proximity of the g.s. to the three-body core +  $n$  +  $n$  decay threshold.

iii)  $\mathcal{E}1$  transitions which play the main role in electromagnetic dissociation are very sensitive to: 1) the structure of the ground state structure and to a lesser extent of

the continuum; 2) the method of treating the Pauli principle; and (not addressed here) possible mechanisms for the  $^{11}\text{Li}$  dynamics during the reaction process.

iv) Only the H-scenario is in a good qualitative agreement with the experimental  $\mathcal{E}1$  deconvoluted response. It should be noted, however, that in all three cases the Pauli principle was taken into account in an approximate way. The question of how to handle the Pauli principle in halo nuclei is thus still open, and one we are still pursuing.

We will return to the nuclear monopole mode and the quadrupole excitation in a larger paper, where also sum rules are discussed in more detail.

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## FIGURES

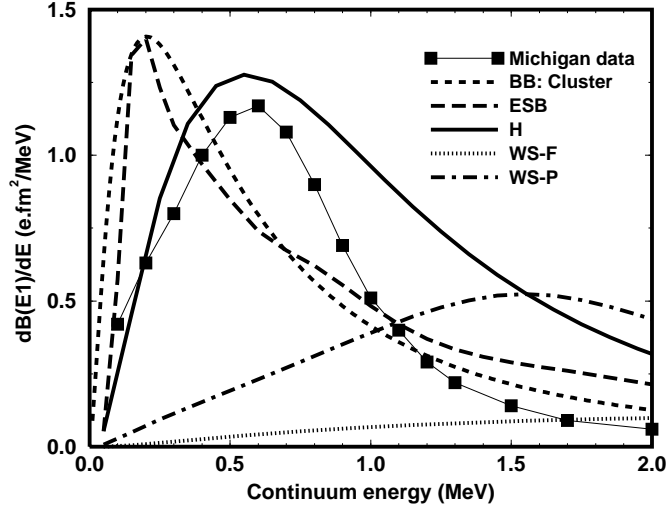


Figure 1: Electric dipole response for  $^{11}\text{Li}$  from the MSU experiment [1] (squares), with the Greens function [8] (ESB, long dashed) and cluster model [7] (BB, short dashed) calculations. The curves H (solid), WS-F (dotted), WS-P (dot-dashed) are the predictions from the current 3-body scenarios.

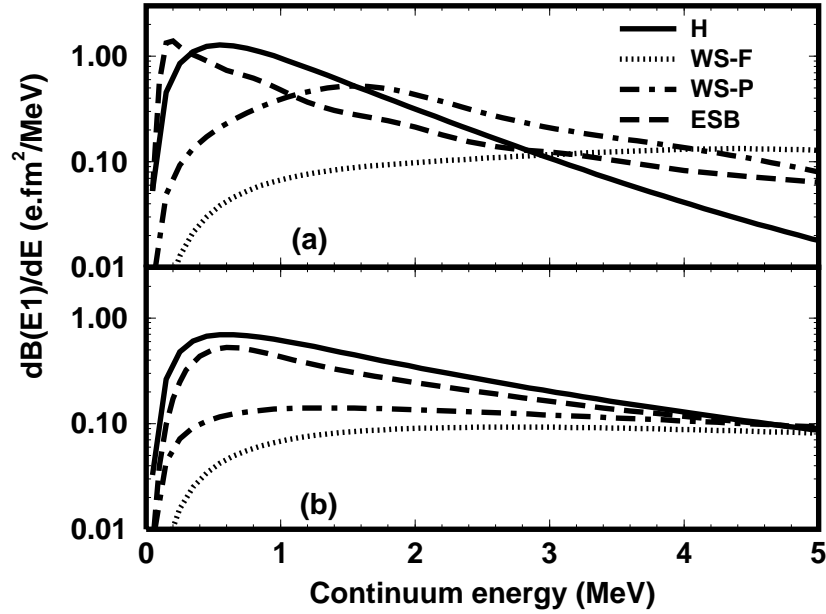


Figure 2: Electric dipole response with correlated (a) and uncorrelated (b) continuum wave functions, for  $^{11}\text{Li}$  in the H, WS-F & WS-P scenarios, and the ESB calculation [8].

## Tables

| K  | L   | S | $l_{nn}$ | $l_{(nn)c}$ | WS-F   | WS-P   | H       |
|--|-----|---|----------|-------------|--------|--------|---------|
| 0  | 0   | 0 | 0        | 0           | 1.17 % | 2.69 % | 96.25 % |
| 2  | 0   | 0 | 0        | 0           | 40.93  | 94.55  | 2.48    |
| 2  | 1   | 1 | 1        | 1           | 54.21  | 0      | 0       |
| 4  | 0   | 0 | 0        | 0           | 1.30   | 1.44   | 0.36    |
| 4  | 1   | 1 | 1        | 1           | 0.19   | 0      | 0       |
| 4  | 0   | 0 | 2        | 2           | 1.88   | 0.29   | 0.66    |
| $\geq 6$                                     | all |   |          |             | 0.31   | 1.02   | 0.22    |
| $E_s$ , MeV                                  |     |   |          |             | 0.332  | 0.248  | 0.295   |
| $r_{mat}$ , fm                               |     |   |          |             | 2.97   | 3.00   | 3.31    |
| $\langle  \rho^2  \rangle$ , fm <sup>2</sup> |     |   |          |             | 48.3   | 48.0   | 76.3    |
| $\langle  \rho^4  \rangle$ , fm <sup>4</sup> |     |   |          |             | 4129   | 4936   | 16135   |
| $\hat{E}_{\mathcal{E}1}$ , MeV               |     |   |          |             | 4.7    | 2.8    | 1.6     |

Table 1: Partial norms, matter radii, binding energies for  $^{11}\text{Li}$  g.s. in WS-P, WS-F and H scenarios, and mean energies for  $\mathcal{E}1$  responses.  $K$  is the hypermoment quantum number, while  $l_{nn}$  and  $l_{(nn)c}$  are orbital angular momenta in the  $n - n$  and  $(nn) - ^9\text{Li}$  degrees of freedom, and couple up to  $L$ .

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